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JET MISSING TRANSVERSE MOMENTA FOR QUARK-GLUON PLASMA THERMOMETRY

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ABSTRACT

At the LHC, where hard multi-jet events are common, large imbalances in the total jet-p_T can arise due to the interaction of jets with the plasma formed in heavy-ion collisions. We find that this missing-p_T spectrum yields a measurement of the initial temperature of the plasma. Using commonly accepted guesses for this temperature, and ISAJET estimates for jet cross sections, we predict an order of thousand events at LHC with unbalanced $p_T > 50$ GeV.

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For the heavy-ion collider mode of the LHC, with $\sqrt{S} = 5.5$ TeV, the cross section for multi-jet events in which none of the jets is less energetic than, say, 50 GeV is of the order of several μb. In a year's run one expects to see about 10^6–10^7 jet events. Of these about 10^4 events contain four or more jets. The expected distribution in the number of jets, N, is shown in Fig. 1. Since the underlying event comes with a multiplicity density, dN_π/dy, of 2000–3000 pions per unit of rapidity, the transverse energy within a cone of $\Delta R = 0.5$ is about 20–30 GeV (assuming $\langle p_T \rangle \approx 0.5$ GeV per pion). Thus 50 GeV jets are easily observed above this background.

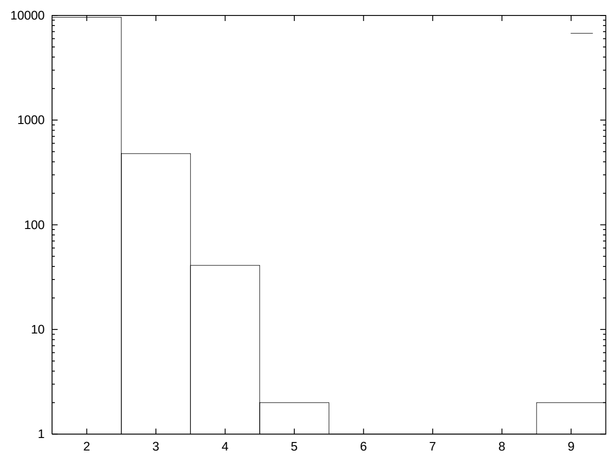

Figure 1

The distribution in the number of jets obtained from a sample of 10^5 events generated in ISAJET with the kinematics mentioned in the text.

If simultaneously, a quark-gluon plasma is formed, then the evolution of the jet is coupled to the plasma through strong interactions. This opens up the possibility of measuring plasma properties using jets. Such measurements could truly be called hard probes of the plasma. The simplest measurements are kinematic, involving changes in the 4-momentum of jets as they evolve through the plasma. It should be emphasised that these are strictly *probes* and not *signals* of the plasma.

There is a history of related studies for dijet events. The first suggestion [1] was to study acoplanarity in 2-jet events. A static model of the plasma was used to show that small acoplanarities can exist. This was extended in [2] to include the effects of hydrodynamics and a mixed phase. In [3] it was shown that a hadron gas could give numerically similiar results. Acoplanarity is an useful variable only for 2-jet events. Very recently energy-energy correlations in 2-jets events have also been investigated [4]. A parallel line of attack is to study the energy loss of a test parton evolving through the plasma. This gives rise to the observation of jet quenching [5].

In the context of forthcoming LHC experiments, we believe that the simplest measurement is of missing transverse momentum, $\not{p}_T$. This is a standard tool for many

other kinds of physics, and one of the LHC detectors can easily perform this measurement. Such a measurement need not be restricted to dijet events. If an experiment with 4π coverage observes exactly N-jets, with a given $\not{p}_T$, then this measurement can be converted into a statement about the initial temperature, T_0, of the plasma. The experimental analysis is made robust by three pleasant aspects of the signal.

i) The $\not{p}_T$-spectrum is almost independent of the initial jet energies when each jet is harder than 50 GeV. Thus there is no need to guess the energy of the jet at the hard vertex, before its interaction with the plasma.

ii) The $\not{p}_T$-spectra for different N are related. Taken together, they provide a cross check for the physical origin of the missing momentum. Unlike acoplanarity, this measurement is not restricted to the dijet sample.

iii) Using well-accepted guesses for T_0 at LHC, we estimate about a thousand events with $\not{p}_T > 50$ GeV. This would yield an accurate experimental measurement of T_0.

Backgrounds coming from lack of full 4π coverage, contamination by the underlying event, and the production of neutrinos and non-standard-model particles can be made small, and will be discussed at the end of this letter.

In this letter we outline the computation of the $\not{p}_T$-spectrum, and show the relation between the $\not{p}_T$-spectra for different number of jets. After this we proceed to display our results. We find that the $\not{p}_T$-spectrum is insensitive to the initial energy in the jet, but strongly sensitive to T_0. It is to be emphasised that $\not{p}_T$ arises when considering only the jets. If the p_T of all hadrons in each collision were to be added up, there would be no $\not{p}_T$.

The Computation: The partons which give rise to jets are produced through a hard scattering in the earliest times of heavy-ion collisions. In the absence of a medium, the measurement of the momenta of the particles constituting the jet would give the momenta of these partons. This enables us to compute jet production cross sections in perturbative QCD. In the presence of a medium, final state scattering would occur. If the medium is a quark gluon plasma, then the individual scatterings may again be computed in perturbative QCD.

The energy of the constituents of the plasma are typically of the order of a GeV, whereas we shall consider jets carrying energy of order 50 GeV. The difference in the scales translates to the kinematic statement that the fractional momentum change of the jet in each scattering is small. Hence, the individual scatterings may be considered independent. We can then write the $\not{\mathbf{p}}_T$-spectrum in an N-jet process as the convolution of many re-scattering probabilities, F_i,—

$$\mathcal{P}_N(\not{\mathbf{p}}_T) \equiv \frac{1}{\sigma}\frac{d^2\sigma}{d^2\not{p}_T} = \sum_{n_1,\cdots,n_N}\prod_{\alpha=1}^{N}\frac{1}{n_\alpha!}\int\left\{\prod_\mu d^2\mathbf{q}_\alpha^\mu F(\mathbf{q}_\mu^\alpha)\right\}\delta^2\left(\not{\mathbf{p}}_T - \sum_{\mu,\alpha}\mathbf{q}_\mu^\alpha\right). \quad (1)$$

Here the cross section σ may be considered to be differential in as many other variables as required. The individual distributions, F_i, may also be considered to be functions of these other variables. Note that the distribution in $\not{p}_T$ is solely given in terms of the interaction between the jet and the plasma. The details of the hard scattering process, including the perturbative expansion of the matrix elements, possible resummations of initial state radiation, jet definitions, *etc.*, are all subsumed into the factors of σ.

The probability of generating a missing transverse momentum $\not{\mathbf{q}}_T$ for a hard parton of species j is given by

$$F^{(j)}(\mathbf{q}_T) \;=\; \sum_i \int_{\tau_0}^{\tau_f} d\tau \mathcal{N}_i(\tau) f_i^{(j)}(\not{\mathbf{q}}_T), \tag{2}$$

where the index i runs over the species of partons in the plasma and $\mathcal{N}_i(\tau)$ is the number density of partons of species i at time τ. The hard parton is assumed to interact with the plasma for times $\tau_0 \leq \tau \leq \tau_f$. The initial time, τ_0, will be taken as the thermalisation time of the plasma. The final time, τ_f, can be obtained from kinematic considerations. The function $f_i^{(j)}$ contains the singular part of the invariant cross section for the scattering of partons of species i and j. Since the relevant jets are gluonic, in the rest of this paper we shall lighten the notation by dropping the index j.

A two-dimensional Fourier transform decouples the δ-function in Eq. (1) and yields the characteristic function of the $\not{p}_T$-distribution as the exponential of the characteristic functions of the distribution F [6]. Thus, after normalisation, we obtain

$$S_N(b) \;=\; \int d^2\not{\mathbf{p}}_T \exp\bigl[-i\mathbf{b}\cdot\not{\mathbf{p}}_T\bigr]\mathcal{P}_N(\not{\mathbf{p}}_T) \;=\; \exp[F_N(b)], \tag{3}$$

where

$$F_N(b) \;=\; 2\pi \sum_{\alpha=1}^{N}\sum_i \int_{\tau_0}^{\tau_f^\alpha} d\tau \mathcal{N}_i(\tau) \int_\mu^{\sqrt{2E^\alpha T(\tau)}} dq q f_i(q)\,[J_0(bq)-1]\,. \tag{4}$$

We have used the fact that f_i is a function only of the magnitude of $\not{\mathbf{q}}_T$ to perform the angular integration in the Fourier transform and obtain the Bessel function $J_0(b\not{q}_T)$. The subtraction in Eq. (4) makes $F(0)=0$, *i.e.*, $S(0)=1$, and hence takes care of the normalisation of $\mathcal{P}_N(\not{\mathbf{p}}_T)$. The lower limit, μ, of the integral over q regulates a leftover logarithmic divergence, and is chosen to be the Debye screening mass evaluated at one-loop order. The $\not{p}_T$-distribution is recovered by an inverse Fourier transform, which, after the angular integration is performed, takes the form

$$\mathcal{P}_N(\not{\mathbf{p}}_T) \;=\; \frac{1}{2\pi}\int db b J_0(b\not{p}_T) S_N(b). \tag{5}$$

Note that many variables are implicit in this formula. The dynamics of the plasma enters through the distributions $\mathcal{N}_i$ and explicitly also through τ_0. The kinematics of the jets enter through the quantities E^α and τ_f^α. We specify these dependences next.

We begin with the evolution of the plasma. In this letter we assume the plasma to be described by longitudinal boost-invariant hydrodynamics. We shall take as our initial conditions a head-on collision of two nuclei of mass number A. The plasma volume is then cylindrically symmetric, with radius $R = r_0 A^{1/3}$ ($r_0 = 6$ GeV^{-1}). A local temperature may be defined, $T(\tau)$. The particle distributions, as functions of this temperature, have an expansion in α_S. To leading order, they are

$$\mathcal{N}_i(\tau) \;=\; g_i T^3(\tau), \qquad \text{where} \qquad g_i \;=\; \begin{cases} \frac{16\zeta(3)}{\pi^2}, & \text{for } i=g \\ \frac{4N_f\zeta(3)}{\pi^2}, & \text{for } i=q,\bar{q} \end{cases} \tag{6}$$

The evolution of the local temperature keeps the entropy density fixed. Hence

$$\tau T^3(\tau) \;=\; \tau_0 T_0^3. \tag{7}$$

T_0 may be related to the pion multiplicity density assuming that there is no entropy generation during hadronisation of the plasma. We can write

$$\tau_0 T_0^3 \;=\; \frac{\pi}{3\zeta(3)R^2}\frac{dN_\pi}{dy}. \tag{8}$$

In most of our calculations, we shall use this formula to specify T_0 in terms of the multiplicity. However, in actual experiments, the jet measurement would, among other things, check the validity of this formula.

The functions f_i also have expansions in α_S. Again we retain the leading term, and obtain

$$f_i(q_T) \;=\; h_i \frac{\alpha_S^2(q_T)}{q_T^4}, \qquad \text{where} \qquad h_i \;=\; \begin{cases} \frac{9}{2}, & \text{for } i=g \\ 2, & \text{for } i=q,\bar{q} \end{cases} \tag{9}$$

From Eqs. (6) and (9) we obtain

$$F(q_T,\tau) \;=\; \lambda \frac{\alpha_S^2(q_T)}{q_T^4}, \qquad \text{where} \qquad \lambda = \frac{36(4+N_f)\zeta(3)}{\pi^2} T^3(\tau). \tag{10}$$

We have used the one-loop order expression for $\alpha_S(q_T)$.

Next, we define the kinematics of the multijet event. Assume the hard vertex to be at distance r from the axis of symmetry. The momentum of the α-th parton can be taken to be

$$p^\alpha \;=\; E^\alpha(1, \mathrm{sech} y^\alpha \cos\theta^\alpha, \mathrm{sech} y^\alpha \sin\theta^\alpha, \tanh y^\alpha), \tag{11}$$

A simple computation now yields

$$\tau_f^\alpha \;=\; \sqrt{r^2 + R^2 - 2rR\cos\phi}, \qquad \text{where} \qquad \sin(\phi - \theta^\alpha) = -\frac{r}{R}\sin\theta^\alpha. \tag{12}$$

Since the polar angles and the position of the hard vertex, r, are not measured, these variables must be averaged over. This is the final ingredient in the computation. We assume that the cross section for a hard process is just the convolution of the hard cross section with a nuclear density function. Assuming an uniform density, we obtain

$$\mathcal{P}_N(\not{p}_T) \;=\; \frac{1}{2\pi}\int dbbJ_0(\not{p}_T b)\int \frac{drr\sqrt{R^2-r^2}d\Omega}{\Omega R^3}S_N(b). \tag{13}$$

The volume element $(d\Omega/\Omega)$ averages over the jet angles θ^α $(\alpha = 1, \cdots, N)$ with uniform measure. The distribution is clearly a function of the energies and rapidities of the jets. The $\not{p}_T$-spectrum is now completely specified.

Some features: A few remarks may help understand the results of the numerical work reported later. In this approach to the interaction between jets and a plasma, each jet undergoes successive scatterings from the soft plasma independently of the others. The net effect is a large imbalance in p_T. This remaining p_T is found in the soft hadronic background underlying the jets, but is invisible in the jet measurement.

Although each single collision of the jet with the plasma only changes the vectorial p_T by a number of the order of T, the number of collisions along the path is large. It is proportional to $\mathcal{N}$ integrated along the path, and hence increases at T_0^3. Note that $\langle \not{p}_T^2 \rangle$ computed from $\mathcal{P}_1(\not{p}_T)$ equals that from $F(\not{p}_T)$. Resummation only changes the shape of the $\not{p}_T$ distribution, introducing a longer tail at the cost of the small-$\not{p}_T$ region.

In [1] QCD initial state radiation was also resummed *a la* Collins and Soper [7] and added to the $\not{p}_T$-distribution. We have not done this for a simple reason. Such a resummation is performed for an N-jet inclusive cross section. For large $\not{p}_T$ this estimates the effect of events where the number of hard jets is larger than N. Since we consider exclusive N-jet events, and large $\not{p}_T$, inclusion of such a term in our formulæ would be erroneous.

In [2] a component of the scattering from the mixed phase was also included. It was observed in [4] that jets escape the plasma before the mixed phase can form. The kinematics in Eq. (12) also lead to the same conclusion. Hence we have not included mixed phase cross sections in our computations. A further ingredient in some studies of the plasma is the inclusion of transverse hydrodynamics. While this decreases the plasma lifetime, it also enhances the plasma volume. Thus the net effect on the $\not{p}_T$-spectrum may be small. It will be studied in more detail in a later paper.

Results: For orientation we display results for a single jet travelling through a plasma. This may arise, for example, as the balancing jet for high-p_T Drell-Yan events. In this case $\not{p}_T$ is the imbalance in p_T between the jet and the lepton pair. For $N = 1$ $d\Omega/\Omega = d\theta/2\pi$. We take $A = 200$, $\tau_0 = 0.1$ and the jet rapidity to be zero. For $dN_\pi/dy = 2000$, the $\not{p}_T$-spectrum is shown in Fig. 2. We obtain $\langle \not{p}_T^2 \rangle = 16$ GeV^2, and find that the distribution goes to zero rapidly near $\not{p}_T = 40$ GeV. The spectra for jet

energies in the range 100 to 1000 GeV are almost equal. The dependence on A at fixed T_0 is small; $\langle \not{p}_T^2 \rangle$ changes by 6% in going from $A = 50$ to $A = 200$. Since this change is due only to the thickness of the plasma firetube, it also demonstrates that we can neglect changes in $\not{p}_T$-distributions for small changes in the impact parameter of the colliding nuclei. Increasing dN_π/dy to 3000 broadens the spectrum a little; $\langle \not{p}_T^2 \rangle = 18$ GeV^2 in this case. However, it is unlikely that such small $\not{p}_T$ can be reliably measured above the soft hadronic background.

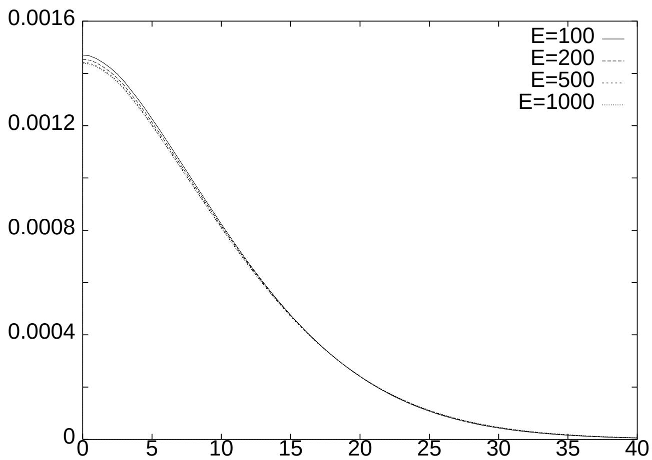

Figure 2

$\mathcal{P}_1(\not{p}_T)$ plotted against $\not{p}_T$ (in GeV) for $dN_\pi/dy = 2000$.

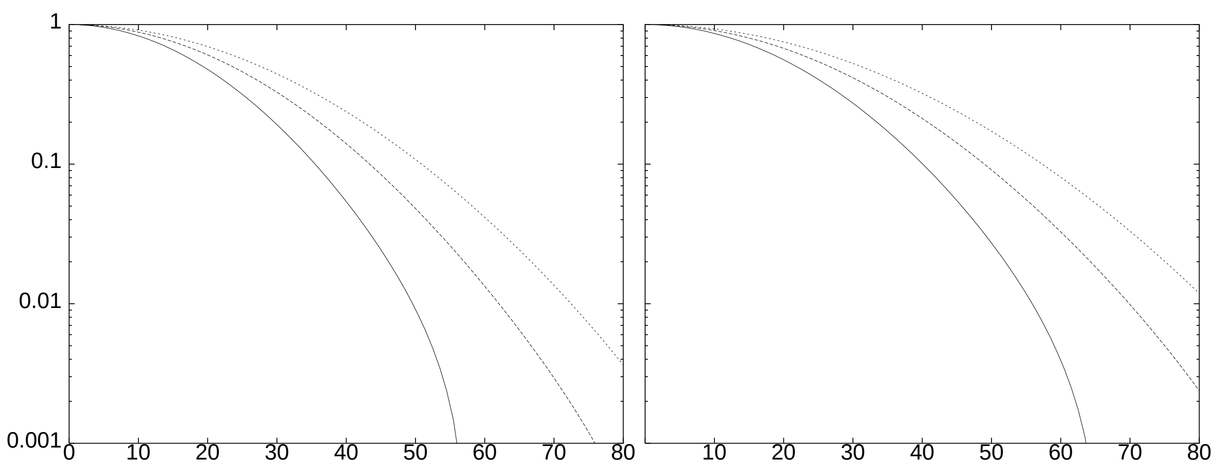

Figure 3

$\mathcal{P}_N^{>}(p_T^{\rm cut})$ plotted against $p_T^{\rm cut}$ (in GeV) for multiplicity densities of (a) 2000 and (b) 3000, per unit of rapidity. The curves are for 2 (full line), 3 (dashed line) and 4 (dotted line) jets at zero rapidity with $\sqrt{\hat{s}} = 400$ GeV.

Multijet events display a much harder $\not{p}_T$-spectrum. At fixed $dN_\pi/dy = 2000$, the values of $\langle \not{p}_T^2 \rangle$ are 23 GeV^2, 29 GeV^2 and 33 GeV^2 for $N = 2$, 3 and 4, respectively. In

Fig. 3 we display

$$\mathcal{P}_{N}^{>}(p_T^{\rm cut}) \;=\; 1 - \int_0^{p_T^{\rm cut}} d\not{p}_T^2 \mathcal{P}_N(\not{\mathbf{p}}_T), \tag{14}$$

i.e., the fraction of events with $\not{p}_T > p_T^{\rm cut}$. For $dN_\pi/dy = 2000$, we see that the fraction of events surviving a cut of $\not{p}_T > 50$ GeV are 1%, 5% and 10% of the total. For comparison, the corresponding numbers increase to 3%, 10% and 17% respectively, when $dN_\pi/dy = 3000$. Thus, the distinction between different values of T_0 is easily observable, and becomes easier with increasing N.

Backgrounds: The main background is the uncertainty in $\not{p}_T$ measurements due to the high multiplicity in the underlying event. If the transverse energy within a jet cone of $\Delta R = 0.5$ due to this effect is E_T^0, then the $\not{p}_T$ uncertainty in an N-jet event becomes $E_T^0\sqrt{N}$. Since E_T^0 is 20 GeV (30 GeV) for $dN_\pi/dy = 2000$ (3000), the $\not{p}_T$ uncertainty ranges from 28 GeV (42 GeV) for $N = 2$ to 40 GeV (60 GeV) for $N = 3$. This background can easily be eliminated by observing distributions with a cut $\not{p}_T > 50$ GeV. As noted in the preceding paragraph, the number of events is still very large.

A second important background comes from limited angular coverage of the detector. Missing p_T could be generated by a jet lying outside the detector. An ISAJET simulation showed that for a detector coverage of $|y| \le 5$ and all polar angles, the background to signal ratio can be reduced to less than 10^{-5} by requiring that the observed jets all lie within $|y| \le 2$.

A third background comes from W or Z production with a subsequent leptonic decay giving a neutrino which escapes undetected. This background is only 4% of the signal, since the cross section is damped by the ratio of the weak and strong couplings times the semi-leptonic branching ratio of the vector boson. It may be reduced even further by vetoing on a hard lepton in the direction of $\not{\mathbf{p}}_T$. Kinematically similiar backgrounds may arise from production of non-standard-model particles. These are small, and one could explore ways of reducing them through topological or kinematic cuts. All these backgrounds will be studied in greater detail in a separate publication.

The suggested experiment: For an LHC detector with full polar angle and $|y| \le 5$ coverage, we suggest the following measurement:

1) Select jet events with all jets harder than 50 GeV and $|y| \le 2$, and bin according to the number of jets N.
2) Veto events with missing transverse momentum less than 50 GeV.
3) Veto events with one or more hard charged leptons in a cone $\Delta R = 0.5$ around the direction of the missing momentum.
4) Match the $\not{p}_T$ distribution against the formulæ in this letter, varying T_0 for each N. The result is a measurement of T_0.

About 10^5 events are expected to pass the cuts in steps 1–3 for an integrated luminosity

of 10^4 $\mu\mathrm{b}^{-1}$. Typically 10% of the events will have $\not{p}_T > 50$ GeV. If heavy-ion collisions give rise to a quark-gluon plasma at the LHC, then in a year of running one could expect to obtain a good measurement of the initial temperature of the plasma.

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